Optimal Hedging Ratios

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Summary

This note examines a simple technique for determining the optimal hedging ratio on foreign assets given a specific portfolio asset allocation. Two methods are derived, one that minimises volatility and one that maximise the ratio of return to volatility. Both methods required estimates of the covariance characteristics of the portfolio and its currency exposure as inputs and two approaches for estimating necessary parameters are discussed: One that uses capital market assumptions for return, volatility and correlation, and one that estimates directly from historical returns on the portfolio. The impact of varying growth and foreign asset exposures is explored, demonstrating that the asset allocation has a large impact of the optimal hedging ratio.

1 Introduction

Determining the level of foreign currency exposure in an investment portfolio is a multi-faceted problem. Holding a large FX position creates currency risk, where domestic purchasing power could be adversely affected by negative currency returns, as explored by Stancu, 2010. Even if long term returns are unaffected, both Schmittmann, 2010 and Stancu, 2010 discuss how large FX exposures can increase portfolio volatility, and Pojarliev et al., 2014 explore how currency exposures can degrade predictability of portfolio performance.

On the other hand, carrying minimal FX exposure requires extremely high hedging ratios, which bring their own issues. Chatsanga and Parkes, 2016 note that high levels of hedging can incur significant costs, including as fees for forward contracts and transaction costs, which can reduce overall portfolio returns. For currencies with negative carry, high levels of hedging can lead to a persistent drag on portfolio performance, as explored by Chakravorty and Awasthi, 2018. Furthermore, Jankensgård and Hagströmer, 2015 note that hedging illiquid assets such as unlisted real assets exposes the fund to a risk of liquidating these assets at a discount during liquidity shortfalls when meeting hedging loss cashflow needs, which can result in severe financial distress and suboptimal outcomes.

This set of trade offs suggest that an *optimal* hedging ratio is one that balances the costs and risks from both extremes. This note examines a simple optimisation approach that takes into account some of the costs and risks outlined: volatility effects and costs of both implementation and carry. Notably the definitions of *optimal* explored ignore considerations of liquidity and currency risk. The only inputs required are capital market assumptions for the asset classes invested, both hedged and unhedged.

2 Derivation

Two expressions for the optimal hedging ratio are derived:

Volatility Minimisation: This method finds the volatility minimising hedging ratio without considerations for hedging costs or predicted returns on currency.

Return-Volatility Maximisation: This method incorporates any assumptions for return on hedging and currency exposure, maximising the ratio between the expectation and volatility of returns.

2.1 Volatility Minimisation

Lets describe portfolio as a collection of assets, both domestic and fully hedged foreign assets, and a exposure to foreign currency, w_c, where w_c is limited to the range between zero and the weight on foreign assets, w_f. The whole portfolio's returns can then be described by:

$$R_p = R_a + w_c R_c \tag{1}$$

where:

- R_a is the returns on the domestic and fully hedged foreign assets
- R_c is the return on currency. The currency in question is a weighted basket based on the currency exposures of the foreign assets.

The variance of the portfolio can be described as:

$$\sigma_p^2 = \sigma_a^2 + w_c^2 \sigma_c^2 + 2 w_c \sigma_{ca} \tag{2}$$

The minimum variance foreign currency exposure can then be found by differentiating σ_p^2 with respect to w_c.

$$\frac{d\sigma_p^2}{dw_c} = 2 w_c \sigma_c^2 + 2\sigma_{ca} \tag{3}$$

Setting the derivative to zero gives the optimal value of w_c

$$w_c = \frac{-\sigma_{ca}}{\sigma_c^2} \tag{4}$$

This value should then be constrained by the bounds on w_c:

$$w_c = \begin{cases} \frac{-\sigma_{ca}}{\sigma_c^2} & \text{if } 0 < \frac{-\sigma_{ca}}{\sigma_c^2} < w_f, \\ 0 & \text{if } \frac{-\sigma_{ca}}{\sigma_c^2} < 0 \\ w_f & \text{if } \frac{-\sigma_{ca}}{\sigma_c^2} > w_f \end{cases} \tag{5}$$

The optimal hedging ratio, h^* can then be calculated, where hedging ratio is the proportion of the foreign asset value that is covered by hedging contracts

$$h^* = 1 - \frac{w_c}{w_f} \tag{6}$$

2.2 Return-Volatility Maximisation

Rather than using volatility alone as the objective function, one can use the ratio of returns to volatility. This is likely preferable in cases where the cost of hedging and/or return on foreign currency is material and predictable.

Using the same setup as before, the return-volatility ratio, S can be expressed as

$$S = \frac{R_a + w_c R_c}{\sqrt{\sigma_a^2 + w_c^2 \sigma_c^2 + 2 w_c \sigma_{ca}}} \tag{7}$$

Differentiating S with respect to w_c gives

$$\frac{dS}{dw_c} = \frac{R_c(\sigma_a^2 + \sigma_{ca} w_c) - R_a(\sigma_c^2 w_c + \sigma_{ca})}{(\sigma_a^2 + w_c(\sigma_c^2 w_c + 2\sigma_{ca}))^{\frac{3}{2}}} \tag{8}$$

Setting to zero and solving for w_c gives

$$w_c = \frac{R_a \sigma_{ca} - R_c \sigma_a^2}{R_c \sigma_{ca} - R_a \sigma_c^2} \tag{9}$$

This result can then be used as in the volatility minimisation case to solve for a hedging ratio, with the applied bounds.

Setting R_c to zero to represent the case of zero return implications from FX exposure causes the formula to simplify to $w_c = \frac{-\sigma_{ca}}{\sigma_c^2}$, the same result as the volatility minimisation approach.

3 Estimation

These methods requires estimating σ_c^2 and σ_{ca}. There are two simple approaches for this, the method that is appropriate depends on what data and assumptions are available.

Covariance Matrix Approach: Used when asset class covariance assumptions are available, likely via CMAs for correlation and volatility.

Regression Approach: Used when returns series for the fully hedged portfolio in question is available. This only works for the volatility minimisation case.

3.1 Covariance Matrix Approach

Let the portfolio of domestic and fully hedged foreign assets be described as $w^T A$ where w are the weights on asset classes A.

We can construct a currency return as a portfolio by creating a long-short portfolio that is long the unhedged asset, short the hedged asset. Ideally the currency exposure of the asset class used should be representative of the currency exposure of the foreign component of the portfolio. This could be achieved either through using a single asset class, such as hedged and unhedged ACWI equities, or by constructing a portfolio of assets that better match the target portfolio. Either way, let these weights be given as z, so the currency returns are described as $z^T A$. The sum of z will be zero.

The required variance and covariance terms are then given by:

- $\sigma_a^2 = w^T \Sigma w$
- $\sigma_c^2 = z^T \Sigma z$
- $\sigma_{ca} = z^T \Sigma w$

These same weights can be used to estimate expected returns if assumptions are also available for these: $R_a = wR$, $R_c = zR$ where R are the assumed ex-ante returns.

These values can be used in the formulas for w_c above.

3.2 Regression Approach

In the volatility minimisation case the optimal currency exposure solution has the structure of covariance divided by variance, the familiar beta formula. This implies that the optimal currency exposure could be calculated as the negative of the beta of the portfolio to its own currency exposure.

Practically this can be estimated by explicitly calculating the covariance and variance terms from historic data, or by fitting a linear regression of the fully hedged portfolio returns on the currency returns

$$R_a = \alpha + \beta R_c + \epsilon \tag{10}$$

$$w_c = -\hat{\beta} \tag{11}$$

To complete the justification for this regression, it is simple to show that the slope coefficient in the regression is equivalent to the beta term we are looking for. The slope in a univariate linear regression, β, is given by the following formula:

$$\beta = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(X_i - \bar{X})^2} \tag{12}$$

This formula can also be written in a more compact form below, which is the beta formula that we are looking for.

$$\beta = \frac{\mathrm{Cov}(X, Y)}{\mathrm{Var}(X)} \tag{13}$$

4 Results

These formulas were tested on some indicative assumptions, detailed in Figure 1 and Figure 2.

Asset Class	Return	Volatility	Proxy
Australian Equities	5.0%	14.8%	ASX:IOZ
Global Equities, Unhedged	7.0%	14.8%	ASX:IWLD
Global Equities, Hedged	7.1%	18.6%	ASX:IHWL
Australian Bonds	4.1%	5.5%	ASX:IGB
Global Bonds, Hedged	4.0%	5.5%	AGG

Figure 1: Return and volatility assumptions used. Return assumptions are simply indicative values inspired by publicly available CMAs from Blackrock, with an illustrative 10bps of carry added to the hedged global equity return. Volatility was measured by annualising rolling weekly returns from the ETFs used as asset class proxies. Price data sourced from Google finance.

Correlation	Australian Equities	Global Equities, Unhedged	Global Equities, Hedged	Australian Bonds	Global Bonds, Hedged
Australian Equities	1.00	0.66	0.78	0.19	0.19
Global Equities, Unhedged	0.66	1.00	0.82	0.12	0.09
Global Equities, Hedged	0.78	0.82	1.00	0.09	0.20
Australian Bonds	0.19	0.12	0.09	1.00	0.61
Global Bonds, Hedged	0.19	0.09	0.20	0.61	1.00

Figure 2: Correlation assumptions used. Correlation was measured from rolling weekly returns on the ETFs used as asset class proxies. Price data sourced from Google finance.

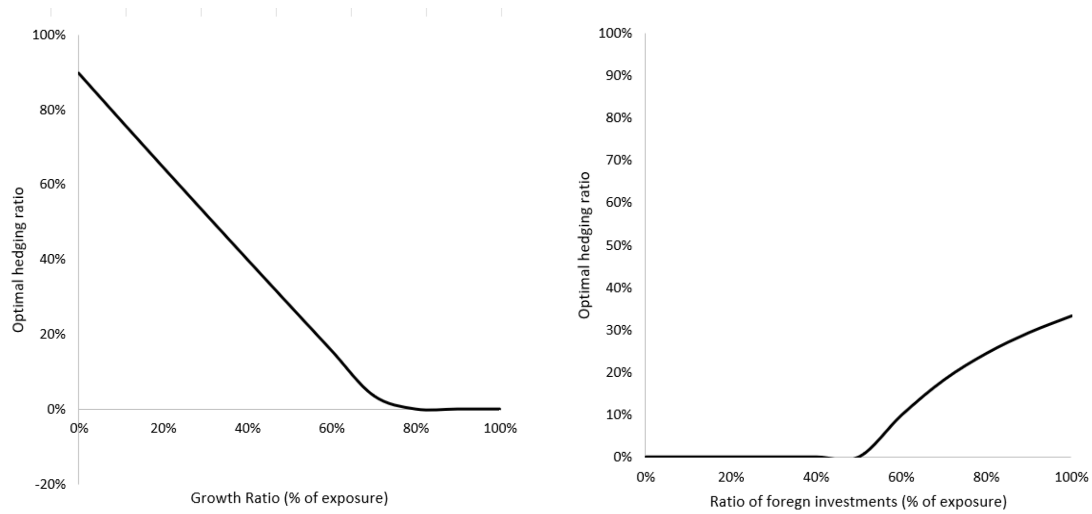
For a sample 60/40 portfolio the results are shown in Table 1. As expected, the slight positive return from hedging increases the optimal hedging ratio when in the return-volatility maximisation method is used.

Method	Optimal FX exposure (w_c)	Optimal hedging ratio
Volatility Minimisation	67%	14%
Return-Volatility Maximisation	56%	16%

Table 1: Results for a 60/40 portfolio, with both bonds and equities split between domestic (Australia) and global in a 1:2 ratio.

Holding the split between foreign and domestic assets constant, we can vary the growth allocation to understand how the optimal hedging ratio changes, as in Figure 3a. As the allocation to equities increases the optimal currency exposure increases, decreasing the hedging ratio from 90% to 0%.

We can also hold the growth allocation constant and vary the allocation between foreign and domestic assets, as in Figure 3b. The optimal weight to currency starts at 35% when there is no foreign asset allocation, and increases linearly to 67% when 100% of assets are foreign. This results in a hedging ratio that initially increases rapidly from zero as global allocation increases, before slowly approaching 33%.



(a) Varying growth allocations, with domestic:global allocation held constant at 1:2.

(b) Varying of domestic-global allocations, with growth allocation held constant at 60%.

Figure 3: Optimal hedging ratio under the return-volatility maximisation method

Note:

Views expressed are the author's, and may differ from those of JANA investments. This material does not constitute investment advice and should not be relied upon as such. Investors should seek independent advice before making investment decisions. Past performance cannot guarantee future results. The charts and tables are shown for illustrative purposes only.

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