Projection-Based Methods for Achieving and Diagnosing Factor Neutrality

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Summary

This note presents two interrelated techniques based on projection matrices for active portfolio management. First, it shows how to derive how to use a projection matrix to filter a view or weight vector so that the resulting portfolio is neutral with respect to a given set of systematic factors. Second, it shows how to invert the process to decompose an existing portfolio's holdings into a benchmark (passive) component, systematic (factor or sector) tilts, and an idiosyncratic (stock-specific) residual.

1 Factor Neutrality via Projection

Assume we have a set of N stocks and a factor model with k systematic factors represented by the $N \times k$ matrix F. Each column of F contains the factor loadings for a specific factor across the N stocks.

A projection matrix that projects any vector onto the column space of F is given by:

$$P_F = F\,(F'F)^{-1}F'.$$

For any vector $x \in R^N$, $P_F x$ is the component of x in the span of F. Therefore, the component orthogonal to the factor space is:

$$x_\perp = x - P_F x = \big(I - F\,(F'F)^{-1}F'\big)x.$$

Defining

$$P = I - F\,(F'F)^{-1}F',$$

we see that P is the projection matrix onto the space orthogonal to the column space of F. In other words, Px extracts the part of x that is independent of the factors in F.

If you have a view vector V that represents forecasted excess returns (or any signal) for each stock, applying the projection matrix gives:

$$V_{\text{idiosyncratic}} = P\,V,$$

which isolates the portion of your view that is uncorrelated with the systematic factors. Similarly, if you have a portfolio weight vector W that may contain unwanted factor exposures, projecting W yields:

$$W_{\text{idiosyncratic}} = P\,W.$$

This "filtered" weight vector is the pure stock-picking component with zero exposure to the factors in F.

1.1 Scaling Factor-Neutral Weights to Meet a Tracking Error Target

Suppose you have already constructed a factor-neutral active weight vector W^* and you wish to adjust its risk such that the portfolio achieves a target tracking error, $\text{TE}_{\text{target}}$. Let Σ be the covariance matrix of stock returns. The tracking error of the active portfolio is:

$$\text{TE}(W^*) = \sqrt{W^{*\prime}\Sigma W^*}.$$

To scale the active bets, introduce a scalar multiplier α so that the new active weights become:

$$W_a = \alpha\, W^*.$$

We choose α to satisfy:

$$\alpha\sqrt{W^{*\prime}\Sigma W^*} = \text{TE}_{\text{target}}, \qquad \alpha = \frac{\text{TE}_{\text{target}}}{\sqrt{W^{*\prime}\Sigma W^*}}.$$

In many practical cases, the overall portfolio is constructed as a combination of a baseline (or benchmark) portfolio W_0 and an active tilt W^*. Then the final portfolio becomes:

$$W = W_0 + \alpha\, W^*.$$

2 Inverse Operation: Decomposing a Portfolio's Weights

Given an existing portfolio with weight vector W, we may decompose W into its systematic and idiosyncratic components. Suppose W_b represents the benchmark weights.

1. Active Component: $A = W - W_b$.
2. Systematic (Factor) Component: $A_{\text{factor}} = P_F\, A = F(F'F)^{-1}F'A$.
3. Idiosyncratic Component: $A_{\text{idiosyncratic}} = A - A_{\text{factor}} = (I - P_F)A$.

Thus, the portfolio can be written as:

$$W = W_b + A_{\text{factor}} + A_{\text{idiosyncratic}}.$$