Covariance Risk Metrics Guide

Grant Holtes

February 28, 2025

Summary

This note provides a concise guide to key ex ante portfolio metrics derived from portfolio and benchmark weights, asset return covariances, and expected returns. It covers foundational measures such as expected return, volatility, and Sharpe ratio, as well as more advanced metrics including tracking error, beta, risk contributions, and diversification ratios. The formulas presented are intended as practical tools for portfolio analysis and risk attribution.

1 Introduction

This document provides a high-level guide for ex ante portfolio metrics computed from:

- The vector of portfolio weights, w
- The vector of benchmark weights, b
- The asset return covariance matrix, Σ
- The vector of asset expected returns, μ

Metrics are grouped by category: Return and Risk Basics, Active Risk and Performance, Beta and Correlation, Risk Decomposition and Attribution, Diversification and Other Metrics, and Beta to a Specific Asset.

2 Return and Risk Basics

2.1 Portfolio Expected Return

Definition: The weighted average of asset expected returns, representing the overall expected return of the portfolio.
Alternative Names: Expected Portfolio Return.

$$E(R_p) = w^T \mu$$

2.2 Active (Excess) Return

Definition: The difference between the portfolio's expected return and the benchmark's expected return.
Alternative Names: Alpha.

$$\alpha = E(R_p - R_b) = (w - b)^T \mu$$

2.3 Portfolio Variance

Definition: A measure of the total risk of the portfolio, expressed as the variance of returns.

$$\sigma_p^2 = w^T \Sigma w$$

2.4 Portfolio Volatility

Definition: The standard deviation of portfolio returns, quantifying overall risk.
Alternative Names: Standard Deviation.

$$\sigma_p = \sqrt{w^T \Sigma w}$$

2.5 Portfolio Excess Volatility

Definition: The additional volatility of portfolio returns compared to the benchmark.

$$\Delta\sigma_{p,b} = \sqrt{w^T \Sigma w} - \sqrt{b^T \Sigma b}$$

3 Active Risk and Performance

3.1 Tracking Error Variance

Definition: The variance of the active (excess) return, i.e., the deviation of the portfolio returns from the benchmark returns.

$$\sigma_{TE}^2 = (w - b)^T \Sigma (w - b)$$

3.2 Tracking Error (Active Risk)

Definition: The standard deviation of the active return, representing the portfolio's active risk Grinold and Kahn, 2000.

$$\sigma_{TE} = \sqrt{(w - b)^T \Sigma (w - b)}$$

Computing Tracking Error for a Composite Portfolio Relative to a Benchmark:
Assume that the covariance matrix Σ represents the covariances among both the individual portfolios and the benchmark. The steps to compute the tracking error are as follows:

1. **Define the Composite Weights:** Let w denote the vector of weights for the individual portfolios in the composite portfolio.
2. **Set the Benchmark Weight:** Construct a benchmark weight vector b such that the benchmark is given a weight of 1 (with the other entries set to 0, if applicable).
3. **Compute the Tracking Error:** Use the tracking error formula as above.

3.3 Information Ratio (IR)

Definition: The ratio of active return (excess return) to tracking error, measuring risk-adjusted active performance.

Alternative Names: Active Risk Adjusted Return.

$$IR = \frac{\alpha}{\sigma_{TE}} = \frac{(w-b)^T \mu}{\sqrt{(w-b)^T \Sigma (w-b)}}$$

3.4 Portfolio Sharpe Ratio

Definition: The ratio of the portfolio's excess return over a risk-free rate to its total risk, as per Grinold and Kahn, 2000.

$$SR = \frac{w^T \mu - r_f}{\sqrt{w^T \Sigma w}}$$

4 Beta and Correlation

4.1 Portfolio Beta with Respect to Benchmark

Definition: Measures the sensitivity of the portfolio return to changes in the benchmark return. Substituting the benchmark for a 100% allocation to a single asset will yield the portfolio beta to that asset.

$$\beta_{p,b} = \frac{w^T \Sigma b}{b^T \Sigma b}$$

4.2 Correlation Between Portfolio and Benchmark

Definition: The correlation coefficient quantifying the linear relationship between the portfolio and benchmark returns.

$$\rho_{p,b} = \frac{w^T \Sigma b}{\sigma_p \, \sigma_b}$$

5 Risk Decomposition and Attribution

5.1 Marginal Contribution to Risk (MCTR)

Definition: The sensitivity of the portfolio risk to a small change in an individual asset's weight Maillard et al., 2010 Roncalli, 2013.
Alternative Names: MCTR.

$$MCTR_i = \frac{\partial \sigma_p}{\partial w_i} = \frac{(\Sigma\, w)_i}{\sigma_p}$$

5.2 Risk Contribution (RC)

Definition: The proportion of total portfolio variance contributed by asset i.

$$RC_i = \frac{w_i\,(\Sigma\, w)_i}{w^T \Sigma w}$$

5.3 Marginal Contribution to Tracking Error (MCTE)

Definition: The sensitivity of the tracking error to a change in asset i's weight.

$$MCTE_i = \frac{(\Sigma\,(w-b))_i}{\sigma_{TE}}$$

5.4 Active Risk Contribution (ARC)

Definition: The contribution of asset i to the overall active risk (tracking error) of the portfolio.

$$ARC_i = \frac{(w_i - b_i)\,(\Sigma\,(w-b))_i}{(w-b)^T \Sigma (w-b)} = \frac{(w_i - b_i)\,(\Sigma\,(w-b))_i}{\sigma_{TE}^2}$$

5.5 Effective Number of Bets

Definition: A measure of diversification of risk contributions in the portfolio. Lower values indicate concentration of risk.
Alternative Names: Effective Number of Risk Factors or Bets.

$$N_{eff} = \frac{1}{\sum_i RC_i^2}$$

6 Diversification and Other Metrics

6.1 Diversification Ratio (DR)

Definition: Compares the weighted average of individual asset volatilities to the overall portfolio volatility, with higher values indicating better diversification, as defined by Choueifaty and Coignard, 2012.

$$DR = \frac{\sum_i w_i \sigma_i}{\sigma_p} \quad \text{where} \quad \sigma_i = \sqrt{\Sigma_{ii}}$$

6.2 Covariance Between Portfolio and Asset i

Definition: The contribution of asset i to the portfolio's covariance structure.

$$\text{Cov}(R_p, R_i) = (w^T \Sigma)_i$$

Note:

Views expressed are the author's, and may differ from those of JANA investments. This material does not constitute investment advice and should not be relied upon as such. Investors should seek independent advice before making investment decisions. Past performance cannot guarantee future results. The charts and tables are shown for illustrative purposes only.

References

Choueifaty, R., & Coignard, É. (2012). On the maximum diversification principle. *Financial Analysts Journal*, *68*(2), 29–41.

Grinold, R. C., & Kahn, R. N. (2000). *Active portfolio management: A quantitative approach for producing superior returns and controlling risk* (2nd). McGraw-Hill.

Maillard, D., Roncalli, T., & Teiletche, P. (2010). On the properties of equally weighted risk contribution portfolios. *The Journal of Portfolio Management*, *36*(4), 60–70.

Roncalli, T. (2013). *Introduction to risk parity and budgeting*. Chapman; Hall/CRC.