Forecasting Returns from Hedging Foreign Assets

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May 21, 2024

Summary

This paper provides a high-level theoretical overview on how to calculate the expected returns on a hedged foreign investments, focusing on the impact of hedging on returns. An expression for hedging returns given an arbitrary hedging ratio is developed, and the impacts of specific assumptions are explored through this theoretical framework. The components of hedging return are decomposed, and their values are empirically estimated within a specific illustrative scenario of an Australian investor hedging a US dollar-denominated asset.

1 Introduction

Forecasting the impact of hedging on a foreign investment is an important part of portfolio design and optimisation. While hedging largely removes the impact of fluctuations in the exchange rate, estimating the impact of the hedge itself on asset returns is non-trivial.

2 Hedging mechanics

For all methods it is assumed a money market hedge, forward exchange rate contracts or an equivalently priced product is used to hedge the foreign investment. Robert Feenstra, 2008 defines how these contracts are priced based on a no-arbitrage condition that the return on domestic deposits, i_d, is equal to the return on foreign deposits, i_f, under the forward rate $\frac{F}{P}(1 + i_f) - 1$. This is the covered interest rate parity condition and it implies a forward rate as in Equation 1.

$$F = S\frac{1 + i_d}{1 + i_f} \tag{1}$$

where:

- i_d is the domestic interest rate
- i_f is the foreign interest rate
- S is the spot exchange rate[1]
- F is the forward exchange rate

[1] Note that all exchange rates are expressed in direct quotation (foreign units per domestic units).

More directly, the hedging return r_h on the hedged value can be given by Equation 2. This hedging return is known with certainty at the start of the hedging period.

$$r_h = \frac{1 + i_d}{1 + i_f} - 1 \tag{2}$$

This can be approximated as in Equation 3, which provides a good approximation when the difference between i_d and i_f is small.

$$r_h \approx i_d - i_f \tag{3}$$

If the hedged value, V_h, is the invested value, V, and the hedged asset is not simply foreign cash or equivalent, then there is likely to exist a difference between the hedged value and the value of the asset at the end of the hedging period. This creates an unhedged component of the investment, V_{uh}, which is subject to fluctuations in the exchange rate. The value of this unhedged component at the end of the hedging period is given by $r_c V_{uh}$, where r_c is the return on the currency. This creates an additional return, r_q, as in Equation 4.

$$r_q = \frac{r_c V_{uh}}{V} \tag{4}$$

As V_{uh} is by definition the difference between the invested amount and the value of the asset at the end of the investing period, $\frac{V_{uh}}{V_h}$ is equal to the return on the asset, r_a. This simplifies the expression for r_q.

$$r_q = r_c r_a \tag{5}$$

Overall the expected return on the hedged investment is given by Equation 6, which adds the expected return on the underlying asset in the foreign currency, r_a, the return on the hedging contract, r_h, and, the return on the unhedged component, r_q.[2]

$$E[r] = E[r_a] + r_h + E[r_c r_a] \tag{6}$$

As described in Park, 2018 this can be expanded to Equation 7.

$$E[r] = E[r_a] + r_h + E[r_c]E[r_a] + cov(r_c, r_a) \tag{7}$$

2.1 Zero expected currency return case

A simplifying assumption is to assume that exchange rates are completely unpredictable and evolve as a random walk, so the expected return on the currency, r_c is zero. This simplifies Equation 7 to Equation 8.

$$E[r] = E[r_a] + r_h + cov(r_c, r_a) \tag{8}$$

[2]It is worth noting that this uses the approximation that $(1 + r_1)(1 + r_2) \approx r_1 + r_2$ for small values of r_1, r_2.

3 Arbitrary hedging ratios

So far we have made the assumption that the hedged value, V_h, is the invested value, V. We can expand the specification to cater to any hedging ratio to explore how hedging ratio impacts expected returns.

$$E[r_q] = E[r_c \frac{V_{uh}}{V}] \tag{9}$$

Given a hedging ratio, h, expressed a percentage of the invested amount, we can express the hedged and unhedged values at the end of the investing period as $V_h = hV$ and $V_{uh} = (1 - h + r_a)V$.

$$E[r_q] = E[r_c \frac{(1 - h + r_a)V}{V}] \tag{10}$$

$$E[r_q] = E[r_c(1 - h + r_a)] \tag{11}$$

$$E[r_q] = E[r_c]E[(1 - h + r_a)] + cov(r_c, (1 - h + r_a)) \tag{12}$$

$$E[r_q] = (1 - h)E[r_c] + E[r_c]E[r_a] + cov(r_c, r_a) \tag{13}$$

The cost of this hedge in return is the cost of hedging multiplied by the hedging weight. Adding in this cost of hedging and the expected asset return we get Equation 14.

$$E[r] = E[r_a] + hr_h + (1 - h)E[r_c] + E[r_c]E[r_a] + cov(r_c, r_a) \tag{14}$$

Equation 14 can be broken down into components:

- $E[r_a]$ - Return from the underlying foreign asset
- hr_h - Cost of hedging
- $(1 - h)E[r_c]$ - Currency return on the unhedged portion of the original asset value
- $E[r_c]E[r_a]$ - Currency return on the unhedged foreign capital gain on the asset
- $cov(r_c, r_a)$ - Correlated movement between asset and currency value

3.1 Hedging the expected value of the investment

A logical choice for the hedging ratio is to choose the ratio that will hedge the expected value of the investment, $h = (1 + E[r_a])$. Substituting into Equation 14 gives the simpler form in Equation 15. Notably the choice of hedging ratio removes the currency return components and leaves a form that is very similar to the case of zero expected currency return, although the hedging cost is slightly increased.

$$E[r] = E[r_a] + (1 + E[r_a])r_h + cov(r_c, r_a) \tag{15}$$

4 Empirical results

The equations derived create questions around the magnitude of the various components of hedging return. Of particular interest are:

- r_h - Cost of hedging
- $cov(r_c, r_a)$ - Correlated movement between asset and currency value

These values are estimated for a range of investments to illustrate their relative importance. The example of an Australian investor holding USD denominated assets is used.

4.1 Return from hedging contract

The return from the hedging contract is given by the interest rate difference as in Equation 2, which is plotted in Figure 1, shows that the hedge has provided a return between 4.68% and -1.2% over the time period analysed.

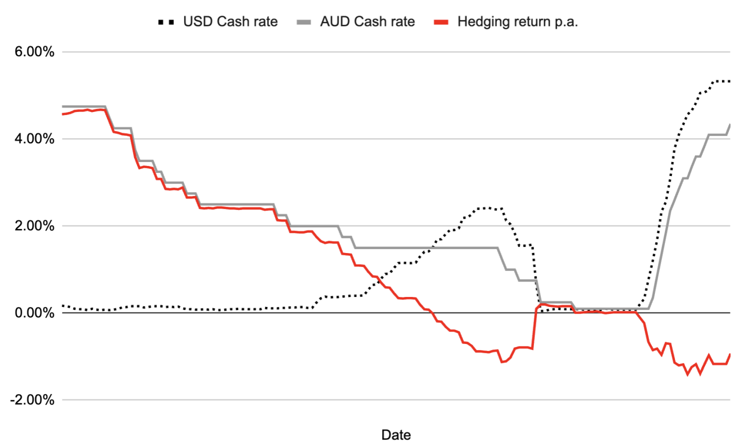

Figure 1: Return from hedging USD as a Australian investor, 2011 to 2023. Central bank cash rates used for illustrative purposes. Data from FRED, 2024 and RBA, 2024

4.2 Covariance term

The covariance term was simply estimated from historical price returns for a small sample of US investments:

SP500 ($SPY): -0.49%

Berkshire Hathaway Inc Class B ($BRK.B): -0.40%

Core US Aggregate Bond ETF ($AGG): -0.08%

These results show that while the covariance term is small, it contributes a material return to an investment. The inverse relationship between US market returns and the USD can be seen in Figure 2, which also illustrates how the magnitude of the covariance term has changed over time.

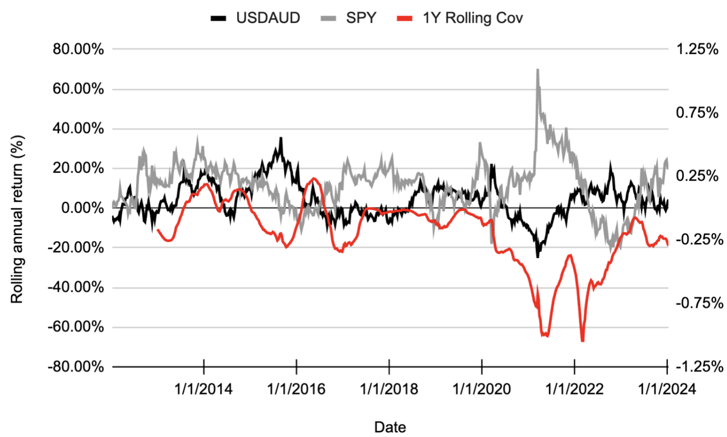

Figure 2: Return on SP500 and the USD (left axis), and rolling covariance between the two (right axis), 2011 to 2023. Price return only, used for illustrative purposes. Data from Google finance

References

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Park, K. (2018). *Fundamentals of probability and stochastic processes with applications to communications*. Springer.

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Robert Feenstra, A. T. (2008). *International macroeconomics*. Worth Publishers.